Making Connections Between and Among Representations in 2016 Mathematics SOL Grades 6 – Algebra II

Virginia Council of Teachers of Mathematics
March 9, 2018

Tina Mazzacane, Mathematics Coordinator
Virginia Department of Education
Office of Science, Technology, Engineering, and Mathematics
Agenda

- Welcome and Introductions
- 2016 Mathematics SOL Implementation Overview
- Effective Mathematics Teaching Practices
- Using and Connecting Math Representations in the 2016 SOL
  - Proportional Relationships to Linear Functions – SOL 6.12 and 7.10
  - Discontinuity – SOL AII.7 (Desmos)
- Support for Implementation
- Closure and Reflection
Welcome and Introductions

• Name
• School Division/School
• Grade Level/Course(s) Taught
2016 Mathematics Standards of Learning Implementation Overview
Implementation of the 2016 Mathematics Standards of Learning - Revised

2017-2018 School Year – Crosswalk Year

- 2009 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning are included in the written and taught curricula.
- Fall 2017 Standards of Learning assessments measure the 2009 Mathematics Standards of Learning and will not include field test items measuring the 2016 Mathematics Standards of Learning.
- Spring 2018 Standards of Learning assessments measure the 2009 Mathematics Standards of Learning and will include field test items measuring the 2016 Mathematics Standards of Learning.

2018-2019 School Year – Full-Implementation Year

- 2016 Mathematics Standards of Learning are included in the written and taught curricula. 2009 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning are included in the written and taught curricula in classrooms administering Fall 2018 Standards of Learning assessments.
- Fall 2018 Standards of Learning assessments, including End-of-Course (Algebra I, Geometry, and Algebra II), will measure the 2009 Mathematics Standards of Learning and include field test items measuring the 2016 Mathematics Standards of Learning.
- Spring 2019 (Grades 3-8 and End-of-Course) Standards of Learning assessments measure the 2016 Mathematics Standards of Learning.
Foci of Revisions

- Improve Vertical Progression of Mathematical Content
- Ensure Developmental Appropriateness of Student Expectations
- Increase Support for Teachers in Mathematics Content
- Ensure Proficiency of Elementary Students in Computational Skills
- Improve Precision and Consistency in Mathematical Language and Format
- Clarify Expectations for Teaching and Learning

2016 REVISIONS
Mathematics Process Goals for Students

“The content of the mathematics standards is intended to support the five process goals for students”
- 2009 and 2016 Mathematics Standards of Learning
2016 SOL Implementation Support Resources

- 2016 Mathematics Standards of Learning
- 2016 Mathematics Standards Curriculum Frameworks
- 2009 to 2016 Crosswalk (summary of revisions) documents
- 2016 Mathematics SOL Video Playlist (Implementation and Resources)
- Narrated 2016 SOL Summary PowerPoints
- 2017 SOL Mathematics Institute Resources
## Grade 6 – Crosswalk (Summary of Revisions): 2016 Mathematics Standards of Learning and Curriculum Framework

### Additions (2016 SOL)
- 6.6a, b – Operations with two integers and solve practical problems [Moved from 7.3]
- 6.9 EKS – Identify regular polygons; draw lines of symmetry for regular polygons
- 6.11b – Determine the effect on measures of center when a value is added, removed, or changed [Moved from 5.16 EKS]
- 6.12 – Represent proportional relationships between two quantities; determine unit rates and complete ratio tables; determine whether a proportional relationship exists; and make connections among representations of proportional relationships
- 6.13 – Solve practical problems with one-step linear equations
- 6.13 EKS – Write verbal expressions and sentences as algebraic expressions and equations; write algebraic expressions and equations as verbal expressions and sentences
- 6.14 – Represent practical situations with inequalities; solve one-step inequalities involving addition and subtraction, and graph solutions on a number line
- 6.14 EKS – Identify a value that is a solution to an inequality

### Deletions from Grade 6 (2009 SOL)
- 6.9 – Ballpark comparisons between U.S. Customary system and metric system of measurements [Included in 7.3 EKS]
- 6.10d – Describe and determine the volume and surface area of a rectangular prism [Included in 7.4a]
- 6.13 – Properties of quadrilaterals [Included in 7.6a]
- 6.15b – Decide which measure of center is appropriate for a given purpose
- 6.16 – Dependent and independent events [Moved to 8.11a]
- 6.16b – Determine probabilities [Included in 8.11b]
- 6.17 – Arithmetic and geometric sequences [Included in AFDA.1 EKS, All.5]

### Parameter Changes/Clarifications (2016 SOL)
- 6.2a EKS – Equivalent fractions limited to denominators of 12 or less or factors of 100
- 6.2b – Compare and order fractions, decimals, and percents extended to positive rational numbers; EKS limited to no more than four; EKS limited to fractions with denominators of 12 or less or factors of 100 to include proper, improper and mixed numbers
- 6.4 EKS – Limitation changed to whole number exponents, versus natural number exponents
- 6.5c EKS – Divisors limited to 3 digit number and decimal divisors limited to hundredths
- 6.6c – Simplify numerical expressions [Moved and modified from 6.8] extended to include integers [EKS extended to include absolute value; exponents limited to 1, 2, and 3 and bases limited to whole numbers; expression may have no more than 3 operations]
- 6.6b EKS - Coordinate values limited to integers
- 6.10a EKS – Number of data values represented on a circle graph limited in order to make comparisons that have denominators of 12 or less or those that are factors of 100
- 6.10c – Compare circle graphs with other graphs now specified as bar graphs, pictographs, and line plots
- 6.11a EKS – Represent mean as a balance point graphically on a line plot
- 6.13 EKS – Solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.
- 6.6, 6.13, 6.14 EKS and US – Apply properties of real numbers and properties of equality/inequality

### Moves within Grade 6 (2009 SOL to 2016 SOL)
- 6.2a – [Moved to 6.2 EKS]
- 6.2 b, c – [Included in 6.2a]
- 6.2d – Compare and order fractions, mixed numbers, decimals and percents [Included in 6.2b]
- 6.4 – [Moved to 6.5a EKS]
- 6.5 – [Moved to 6.4]
- 6.6 – [Moved to 6.5a, b]
- 6.7 – [Moved to 6.5c]
- 6.8 – [Moved to 6.6c and modified]
- 6.10a, b, c – [Moved to 6.7a, b, c]
- 6.11 – [Moved to 6.8]
- 6.12 – [Moved to 6.9]; Draw polygons in the coordinate plane and find side lengths using the coordinates [Moved to 6.8]
- 6.13 – [Moved to 6.12]
- 6.14 – [Moved to 6.10]
- 6.15a – [Moved to 6.11a]
- 6.16 – [Moved to 6.13]
- 6.19 – Investigate and recognize properties [Incorporated into EKS and US for 6.6, 6.13, and 6.14]
- 6.20 – [Moved to 6.14]

**EKS** = Essential Knowledge and Skills, referring to the column on the right side of the Curriculum Framework
**US** = Understanding the Standard, referring to the column on the left side of the Curriculum Framework
### Comparison of Mathematics Standards of Learning – 2009 to 2016

<table>
<thead>
<tr>
<th>2009 SOL</th>
<th>2016 SOL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Number Sense</strong></td>
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</tr>
<tr>
<td><em>On the state assessment, items measuring this objective are assessed without the use of a calculator.</em></td>
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</tr>
</tbody>
</table>

#### 6.1 The student will describe and compare data, using ratios, and will use appropriate notations, such as $\frac{a}{b}$, a to b, and a:b.

#### 6.2 The student will
- a) investigate and describe fractions, decimals, and percents as ratios; [Moved to 6.2 EKS]
- b) identify a given fraction, decimal, or percent from a representation; [Included in 6.2a]
- c) demonstrate equivalent relationships among fractions, decimals, and percents; * and [Included in 6.2a]
- d) compare and order fractions, decimals, and percents. * [Included in 6.2b]

#### 6.3 The student will
- a) identify and represent integers;
- b) order and compare integers; and
- c) identify and describe absolute value of integers.

#### 6.4 The student will demonstrate multiple representations of multiplication and division of fractions. [Moved to 6.5 EKS]

#### 6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

#### 6.6 The student will
- a) multiply and divide fractions and mixed numbers; * and
- b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.

#### 6.7 The student will solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved to 6.5c]

#### 6.2 The student will
- a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; *
- b) compare and order positive rational numbers.*

#### 6.3 The student will
- a) identify and represent integers;
- b) compare and order integers; and
- c) identify and describe absolute value of integers.

#### 6.4 The student will recognize and represent patterns with whole number exponents and perfect squares.

#### 6.5 The student will
- a) multiply and divide fractions and mixed numbers; *
- b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and
- c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7]
Effective Mathematics Teaching Practices
“Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.”

NCTM (2014), *Principles to Action*  
Executive Summary

YOU make all the difference! Thank you!!!
NCTM *Principles to Actions*
*Ensuring Mathematical Success for All*

<table>
<thead>
<tr>
<th>High Leverage Mathematics Teaching Practices</th>
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<td>1. Establish mathematics goals to focus learning.</td>
</tr>
<tr>
<td>2. Implement tasks that promote reasoning and problem solving.</td>
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<tr>
<td>3. Use and connect mathematical representations.</td>
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<tr>
<td>4. Facilitate meaningful mathematical discourse.</td>
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<tr>
<td>5. Pose purposeful questions.</td>
</tr>
<tr>
<td>6. Build procedural fluency from conceptual understanding.</td>
</tr>
<tr>
<td>7. Support productive struggle in learning mathematics.</td>
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<td>8. Elicit and use evidence of student thinking.</td>
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### NCTM Principles to Actions

*Ensuring Mathematical Success for All*

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# Use and Connect Mathematical Representations

<table>
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</tr>
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<tbody>
<tr>
<td><strong>Teacher Actions</strong></td>
</tr>
<tr>
<td>Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
</tr>
<tr>
<td>Introducing forms of representations that can be useful to students.</td>
</tr>
<tr>
<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
</tr>
<tr>
<td>Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
</tr>
<tr>
<td>Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.</td>
</tr>
</tbody>
</table>

Use and Connect Mathematical Representations

The term *representation* refers both to process and to product – in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself.....Moreover, the term applies to processes and products that are observable externally as well as to those that occur “internally” in the minds of people doing mathematics.

- NCTM 2000, p. 67
Teaching Framework for Mathematics

Establish mathematics goals to focus learning.

Implement tasks that promote reasoning and problem solving.

Build procedural fluency from conceptual understanding.

Facilitate meaningful mathematical discourse.

Use and connect mathematical representations.

Pose purposeful questions.

Elicit and use evidence of student thinking.

Support productive struggle in learning mathematics.

Virginia Mathematics Process Goals

1. Representations
2. Connections
3. Problem Solving
4. Reasoning
5. Communication

Using and Connecting Mathematical Representations in the 2016 SOL
Mathematical Representations - 2016 SOL

6.12 The student will
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

7.10 The student will
e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

8.16 The student will
e) make connections between and among representations of a linear function using verbal descriptions, tables, equation, and graphs.
# 2016 Patterns, Functions and Algebra – 6-8

<table>
<thead>
<tr>
<th></th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Relationships/Functions</td>
<td>Represent proportional relationships; unit rates; ratio tables; determine if a proportional relationship exists; and make connections between and among representations</td>
<td>Slope as a rate of change; write an equation in $y = mx$ form to represent a proportional relationship and $y = x + b$ form to represent an additive relationship and graph their lines; and make connections between and among representations</td>
<td>Slope of a line given a graph; slope and $y$-intercept of a linear function and graph using $y = mx + b$ form; and make connections between and among representations</td>
</tr>
<tr>
<td>Solving Equations</td>
<td>Solve one-step equations including within practical problems</td>
<td>Solve two-step equations including within practical problems</td>
<td>Solve multi-step equations (including variables on both sides) including within practical problems</td>
</tr>
<tr>
<td>Solving Inequalities</td>
<td>Represent practical situations with inequalities; solve one-step inequalities using addition/subtraction and graph the solution on a number line</td>
<td>Solve one- and two-step inequalities (all operations) including within practical problems; graph solution on a number line</td>
<td>Solve multi-step inequalities (including variables on both sides) including within practical problems; graph solution on a number line</td>
</tr>
<tr>
<td>Algebraic Expressions</td>
<td>Evaluate algebraic expressions for given replacement values of the variable</td>
<td>Evaluate an algebraic expression for given replacement values of the variable; simplify expressions in one variable</td>
<td></td>
</tr>
</tbody>
</table>
### Algebra (Proportional Reasoning) Progression

<table>
<thead>
<tr>
<th>SOL 6.12</th>
<th>SOL 7.10</th>
<th>SOL 8.16</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical connections, and representations to...</td>
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<td>The student will use problem solving, mathematical communication, mathematical connections, and representations to...</td>
</tr>
<tr>
<td>• Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)</td>
<td>• Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)</td>
<td>• Given a table of values for a linear function, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)</td>
</tr>
<tr>
<td>• Identify the unit rate of a proportional relationship represented by a table, or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>• Identify the unit rate of a proportional relationship represented by a table, or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>• Given a linear function in the form ( y = mx + b ), identify the slope and y-intercept. (b)</td>
</tr>
<tr>
<td>• Determine a missing value in a ratio table or a verbal description, including those represented in a practical situation, and write an equation in the form ( y = mx ) to represent the relationship. Slope will be limited to positive values. (a)</td>
<td>• Determine a missing value in a ratio table or a verbal description, including those represented in a practical situation, and write an equation in the form ( y = mx ) to represent the relationship. Slope will be limited to positive values. (a)</td>
<td>• Given a table of values for a linear function, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)</td>
</tr>
<tr>
<td>• Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, or a set of ordered pairs. Slope will be limited to positive values. (b)</td>
<td>• Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, or a set of ordered pairs. Slope will be limited to positive values. (b)</td>
<td>• Given a linear function in the form ( y = mx + b ), identify the slope and y-intercept. (b)</td>
</tr>
<tr>
<td>• Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>• Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>• Determine the y-intercept, ( b ), in an equation in the form ( y = mx + b ), given an ordered pair on the line and the slope, or given the equation of the line in the form ( y = mx + b ). The coordinates of the ordered pairs shown in the graph will be limited to integers. (b)</td>
</tr>
<tr>
<td>• Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)</td>
<td>• Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)</td>
<td>• Graph a line representing an additive relationship ( y = x + b ), given two ordered pairs on the line and the slope, or given the equation of the line in the form ( y = mx + b ). The y-intercept ( b ) is limited to integer values and slope is limited to 1. (d)</td>
</tr>
<tr>
<td>• Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)</td>
<td>• Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)</td>
<td>• Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e)</td>
</tr>
</tbody>
</table>

#### RATIO TABLES

#### UNIT RATES

#### SLOPE as RATE of CHANGE

#### SLOPE and y-INTERCEPT

\[
y = mx + b
\]

- Given a table of values for a **linear function**, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)
- Given a linear function in the form \( y = mx + b \), identify the slope and y-intercept. (b)
- Given a **linear function** in the form \( y = mx + b \), identify the slope and y-intercept. (b)
- Given the equation of a **linear function** in the form \( y = mx + b \), graph the function. The value of the y-intercept will be limited to integers. (d)
- Make connections between and among representations of a **linear function** using verbal descriptions, tables, equations, and graphs. (e)
Perfect Purple Paint

- Perfect Purple Paint is made by mixing 2 cups blue paint to 3 cups red paint.

- How much of each color would be needed to make a total of 20 cups of Perfect Purple Paint?

Work with a partner to create a model using linking cubes to solve this problem.
Introduction to Ratios and Proportional Relationships


• Using the list of teacher and student actions for Mathematics Teaching Practice #3: Use and Connect Mathematical Representations:
  o Identify teacher actions from the video that promote using and connecting representations?
  o Identify student actions from the video that promote using and connecting representations?
UNIT RATE: 1 ½ CUPS RED PER 1 CUP BLUE
## RATIO TABLE

<table>
<thead>
<tr>
<th>BLUE (cups)</th>
<th>RED (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{2}$ or 1.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

\[
y = \frac{3}{2}x
\]

\[
y = mx
\]
Functions, Discontinuity, and Desmos
Multiple Representations of Functions

A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically including

f) connections between and among multiple representations of functions including verbal descriptions, tables, equations, and graphs.

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

g) connections between and among multiple representations of functions including verbal descriptions, tables, equations, and graphs.
SOL All.7 - Discontinuity

All.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include
a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard

- A function is said to be continuous on an interval if its graph has no jumps or holes in that interval.
- The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.
- Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities.

Essential Knowledge and Skills

- Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. (a, d, e)
- Describe a function as continuous or discontinuous. (a)
Desmos - Open Source Versions

DESMOS ONLINE GRAPHING UTILITY: https://desmos.com/calculator

DESMOS CLASSROOM ACTIVITIES: https://teacher.desmos.com

DESMOS STUDENT ACTIVITY LINK: https://student.desmos.com

LEARN MORE ABOUT DESMOS: https://learn.desmos.com
Desmos – Additional Features in Development

DESMOS GEOMETRY UTILITY (Beta Version):
https://desmos.geometry.com/

DESMOS GRAPHING UTILITY ENHANCEMENTS
https://www.desmos.com/calculator :
DISTRIBUTIONS – adds statistical tests, such as normal cdf and pdf, t-tests, etc. to graphing utility functions

ADDITIONAL ACCESSIBILITY FEATURES
Desmos – Graphing Utility

https://www.desmos.com/calculator

\[ f(x) = 2x - 3 \quad \{x < 2\} \]

\[ f(x) = -4 \quad \{x \geq 2\} \]
Card Sort – Making Connections (SOL 7.10e)

Welcome!

Enter your class code:

I

Join

Sign in to come back to your work later:

Google Sign in with Google or Sign in with Desmos

Hey, students!

Go to student.desmos.com and type in:

24UYH

You can also share this link with your students:

https://student.desmos.com/?prepopulateCo
Implementation Support Resources
Implementation Support Resources

Currently Available

- 2016 Mathematics Standards of Learning
- 2016 Mathematics Standards Curriculum Frameworks
- 2009 to 2016 Crosswalk (summary of revisions) documents
- 2016 Mathematics SOL Video Playlist (Overview, Vertical Progression & Support, Implementation and Resources)
- Progressions for Selected Content Strands (K-5: Number and Number Sense, Computation and Estimation)
- Narrated 2016 SOL Summary PowerPoints
- 2017 SOL Mathematics Institutes Professional Development Resources

Coming in Spring/Summer 2018

- Updated and New Lesson Plans
- Updated Vocabulary Cards
- Virginia Board of Education Textbook Approval List
Closure and Participant Reflection

Exit Ticket – What Stuck?

On a Post-It Note –
This is what stuck with me today........
QUESTIONS?

Tina.Mazzacane@doe.virginia.gov
Mathematics@doe.virginia.gov
### Grade 6 - Algebra II Progression - Proportional Relationships, Functions, and Solving Equations/Inequalities

<table>
<thead>
<tr>
<th>6.12 The student will</th>
<th>7.10 The student will</th>
<th>8.15 The student will</th>
<th>A.6 The student will</th>
<th>All.6 For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will</th>
</tr>
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<tbody>
<tr>
<td>a) represent a proportional relationship between two quantities, including those arising from practical situations.</td>
<td>a) determine the slope, ( m ), as rate of change in a proportional relationship between two quantities and write an equation in the form ( y = mx ) to represent the relationship;</td>
<td>a) determine whether a given relation is a function; and</td>
<td>a) determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line;</td>
<td>a) recognize the general shape of function families; and</td>
</tr>
<tr>
<td>b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;</td>
<td>b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in the form ( y = mx ) form where ( m ) represents the slope as rate of change.</td>
<td>b) determine the domain and range of a function.</td>
<td>b) write the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line;</td>
<td>b) use knowledge of transformations to convert between equations and the corresponding graphs of functions.</td>
</tr>
<tr>
<td>c) determine whether a proportional relationship exists between two quantities; and</td>
<td>c) determine the ( y )-intercept, ( b ), in an additive relationship between two quantities and write an equation in the form ( y = x + b ) to represent the relationship;</td>
<td>Determine whether a relation, represented by a set of ordered pairs, a table, or a graph of discrete points is a function. Sets are limited to no more than 10 ordered pairs.</td>
<td>c) graph linear equations in two variables.</td>
<td>• Recognize the general shape of function families. (a)</td>
</tr>
<tr>
<td>d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.</td>
<td>d) graph a line representing an additive relationship between two quantities given the ( y )-intercept and an ordered pair, or given the equation in the form ( y = x + b ), where ( b ) represents the ( y )-intercept; and</td>
<td>• Determine the slope of a line, given the equation of a linear function. (a)</td>
<td>• Identify the graph of a function from the equation. (b)</td>
<td>• Recognize graphs of parent functions. (a)</td>
</tr>
<tr>
<td>• Make a table of equivalent ratios to represent a proportional relationship between two quantities, given a ratio. (a)</td>
<td>(a)</td>
<td>• Determine the slope of a line, given the graph of a line. (a)</td>
<td>• Write the equation of a function given the graph. (b)</td>
<td>• Identify the graph of a function from the equation. (b)</td>
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<tr>
<td>• Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a)</td>
<td>• Determine the slope of a line, given the coordinates of two points on the line. (a)</td>
<td>• Recognize and describe a line with a slope or rate of change that is positive, negative, or undefined. (a)</td>
<td>• Graph a transformation of a parent function, given the equation. (b)</td>
<td>• Identify the transformation(s) of a function. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. (b)</td>
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<td>• Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>• Determine the slope of a line, given the equation of a linear function. (a)</td>
<td>• Write the equation of a line when given the graph of a line. (b)</td>
<td>• Interpret the transformation(s) of a function. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. (b)</td>
<td>• Investigate and verify transformations of functions using a graphing utility. (a, b)</td>
</tr>
<tr>
<td>• Determine a missing value in a ratio table that represents a proportional relationship between two quantities</td>
<td>• Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.</td>
<td>• Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.</td>
<td>• Write the equation of a line when given two points on the line whose coordinates are integers. (b)</td>
<td>All.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include</td>
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<tr>
<td>• Determine the slope, ( m ), as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form ( y = mx ) to represent the relationship. Slope will</td>
<td>• Determine the independent and dependent variable, given a practical situation modeled by a linear function;</td>
<td>• Recognize and describe a line with a slope that is positive, negative, or zero (0). (a)</td>
<td>• Write the equation of a vertical line as ( x = a ). (b)</td>
<td>a) domain, range, and continuity;</td>
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<tr>
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<th>Algebra I</th>
<th>Algebra II</th>
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<tr>
<td><strong>PROPORTIONAL RELATIONSHIPS/ FUNCTIONS</strong>&lt;br&gt;using a unit rate. Unit rates are limited to positive values. (b)</td>
<td><strong>Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, $m$, as rate of change. Slope will be limited to positive values.</strong> (b)</td>
<td><strong>Given a table of values for a linear function, identify the slope and $y$-intercept. The table will include the coordinate of the $y$-intercept.</strong> (b)</td>
<td><strong>Graph a linear equation in two variables, including those that arise from a variety of practical situations.</strong> (c)</td>
<td><strong>Intervals in which a function is increasing or decreasing;</strong>&lt;br&gt;<strong>c) extrema;</strong>&lt;br&gt;<strong>d) zeros;</strong>&lt;br&gt;<strong>e) intercepts;</strong>&lt;br&gt;<strong>f) values of a function for elements in its domain;</strong>&lt;br&gt;<strong>g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;</strong>&lt;br&gt;<strong>h) end behavior;</strong>&lt;br&gt;<strong>i) vertical and horizontal asymptotes;</strong>&lt;br&gt;<strong>j) inverse of a function; and</strong>&lt;br&gt;<strong>k) composition of functions, algebraically and graphically.</strong>&lt;br&gt;<strong>determine f(x).</strong> (e)</td>
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<tr>
<td><strong>Determine whether a proportional relationship exists between two quantities when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values.</strong> (c)</td>
<td><strong>Determine the $y$-intercept, $b$, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx + b$, where $m$ represents the slope as rate of change. Slope will be limited to positive values.</strong> (b)</td>
<td><strong>Given the equation of a linear function in the form $y = mx + b$, identify the slope and $y$-intercept.</strong> (b)</td>
<td><strong>Identify the domain, range, zeros, and intercepts;</strong>&lt;br&gt;<strong>d) extrema;</strong>&lt;br&gt;<strong>e) intercepts;</strong>&lt;br&gt;<strong>f) values of a function for elements in its domain;</strong></td>
<td><strong>Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities.</strong> (a, d, e)</td>
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<tr>
<td><strong>Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values.</strong> (d)</td>
<td><strong>Graph a line representing a proportional relationship between two quantities given an equation of the line in the form $y = mx$, where $m$ represents the slope as rate of change. Slope will be limited to positive values.</strong> (b)</td>
<td><strong>Given the graph of a linear function, identify the slope and $y$-intercept. The value of the $y$-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers.</strong> (b)</td>
<td><strong>Identify the dependent and independent variable, given a practical situation modeled by a linear function.</strong> (c)</td>
<td><strong>Identify the domain, range, zeros, and intercepts;</strong>&lt;br&gt;<strong>d) extrema;</strong>&lt;br&gt;<strong>e) intercepts;</strong>&lt;br&gt;<strong>f) values of a function for elements in its domain;</strong>&lt;br&gt;<strong>g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;</strong>&lt;br&gt;<strong>h) end behavior;</strong>&lt;br&gt;<strong>i) vertical and horizontal asymptotes;</strong>&lt;br&gt;<strong>j) inverse of a function; and</strong>&lt;br&gt;<strong>k) composition of functions, algebraically and graphically.</strong>&lt;br&gt;<strong>determine f(x).</strong> (e)</td>
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<tr>
<td><strong>Algebra I</strong></td>
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<tr>
<td>• Represent relations and functions using verbal descriptions, tables, equations, and graph. Given one representation, represent the relation in another form. (f)</td>
<td>• For any x value in the domain of f, determine f(x). (f)</td>
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<tr>
<td>• Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f)</td>
<td>• Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g)</td>
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<tr>
<td>• Describe the end behavior of a function. (h)</td>
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<tr>
<td>• Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i)</td>
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<tr>
<td>• Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j)</td>
<td>• Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j)</td>
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<tr>
<td>• Graph the inverse of a function as a reflection over the line y = x. (j)</td>
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<tr>
<td>• Determine the composition of two functions algebraically and graphically. (k)</td>
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<tr>
<td>• Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i, j, k)</td>
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## SOLVING EQUATIONS

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<tbody>
<tr>
<td>6.13 The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.</td>
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<tr>
<td>- Identify examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.</td>
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<td>- Represent and solve one-step linear equations in one variable, using a variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale.</td>
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<tr>
<td>- Apply properties of real numbers and properties of equality to solve a one-step equation in one variable.</td>
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<tr>
<td>- Represent and solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.</td>
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<tr>
<td>- Represent and solve a practical problem with a one-step linear equation in one variable.</td>
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<td>7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.</td>
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<tr>
<td>- Represent and solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable, using a variety of concrete materials and pictorial representations.</td>
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<tr>
<td>- Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable.</td>
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<tr>
<td>- Write algebraic expressions and equations as verbal expressions and sentences.</td>
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<tr>
<td>- Solve practical problems that require the solution of a two-step linear equation.</td>
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<td>8.17 The student will solve multistep linear equations in one variable, including practical problems that require the solution of a multistep linear equation in one variable.</td>
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<tr>
<td>- Solve practical problems that require the solution of a multistep linear equation.</td>
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<td>- Confirm algebraic solutions to linear equations in one variable.</td>
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<tr>
<td>A.4 The student will solve</td>
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<tr>
<td>a) multistep linear equations in one variable algebraically;</td>
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<td>b) quadratic equations in one variable algebraically;</td>
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<td>c) literal equations for a specified variable;</td>
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<tr>
<td>d) systems of two linear equations in two variables algebraically and graphically; and</td>
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<tr>
<td>e) practical problems involving equations and systems of equations.</td>
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<tr>
<td>- Determine whether a linear equation in one variable has one, an infinite number, or no solutions. (a)</td>
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<tr>
<td>- Apply the properties of real numbers and properties of equality to simplify expressions and solve equations. (a, b)</td>
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<tr>
<td>- Solve multistep linear equations in one variable algebraically. (a)</td>
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<tr>
<td>- Solve quadratic equations in one variable algebraically. Solutions may be rational or irrational. (b)</td>
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<tr>
<td>- Solve a literal equation for a specified variable. (c)</td>
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<tr>
<td>- Given a system of two linear equations in two variables that has a unique solution, solve the system by substitution or elimination to identify the ordered pair which satisfies both equations. (d)</td>
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<tr>
<td>- Determine whether a system of two linear equations in two variables that has a unique solution, solve the system graphically by identifying the point of intersection. (d)</td>
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<tr>
<td>- Solve and confirm algebraic solutions to a system of two linear equations using a graphing utility. (d)</td>
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<td>- Determine whether a system of two linear equations has one, an infinite number, or no solutions. (d)</td>
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<tr>
<td>- Write a system of two linear equations that models a practical situation. (e)</td>
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<tr>
<td>- Interpret and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a practical situation. (e)</td>
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<tr>
<td>- Solve practical problems involving equations and systems of equations. (e)</td>
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<tr>
<td>7.14 The student will solve multistep linear equations in one variable using a variety of concrete materials and pictorial representations.</td>
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<tr>
<td>- Confirm algebraic solutions to linear equations in one variable.</td>
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<tr>
<td>8.18 The student will solve multistep linear equations in one variable, including practical problems that require the solution of a multistep linear equation in one variable.</td>
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<tr>
<td>- Write verbal expressions and sentences as algebraic expressions and equations.</td>
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**All.3 The student will solve**

a) absolute value linear equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions.

- Solve absolute value linear equations or inequalities in one variable algebraically. (a) |
- Solve a quadratic equation over the set of complex numbers algebraically. (b) |
- Calculate the discriminant of a quadratic equation to determine the number and type of solutions. (b) |
- Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. (c) |
- Solve an equation containing no more than one radical expression algebraically and graphically. (d) |
- Solve equations and verify algebraic solutions using a graphing utility. (a, b, c, d) |

**All.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.**

- Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables. |
- Solve a linear-quadratic system of two equations in two variables algebraically and graphically. |
- Solve a quadratic-quadratic system of two equations in two variables algebraically and graphically. |
- Solve systems of equations and verify solutions of systems of equations with a graphing utility. |
6.14 The student will
a) represent a practical situation with a linear inequality in one variable; and
b) solve one-step linear inequalities in one variable, involving addition and subtraction, and graph the solution on a number line.
- Given a verbal description, represent a practical situation with a one-variable linear inequality. (a)
- Apply properties of real numbers and the addition or subtraction property of inequality to solve a one-step linear inequality in one variable, and graph the solution on a number line. Numeric terms being added or subtracted from the variable are limited to integers. (b)
- Given the graph of a linear inequality with integers, represent the inequality two different ways (e.g., \( x < -5 \) or \( -5 > x \)) using symbols. (b)
- Identify a numerical value(s) that is part of the solution set of a given inequality. (a, b)

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.
- Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational.
- Represent solutions to inequalities algebraically and graphically using a number line.
- Write verbal expressions and sentences as algebraic expressions and inequalities.
- Write algebraic expressions and inequalities as verbal expressions and sentences.
- Solve practical problems that require the solution of a one- or two-step inequality.
- Identify a numerical value(s) that is part of the solution set of a given inequality.

8.18 The student will solve multistep linear inequalities in one variable, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.
- Apply properties of real numbers and properties of inequality to solve multistep linear inequalities (up to four steps) in one variable with the variable on one or both sides of the inequality. Coefficients and numeric terms will be rational. Inequalities may contain expressions that need to be expanded (using the distributive property) or require collecting like terms to solve.
- Graph solutions to multistep linear inequalities on a number line.
- Write verbal expressions and sentences as algebraic expressions and inequalities.
- Write algebraic expressions and inequalities as verbal expressions and sentences.
- Solve practical problems that require the solution of a multistep linear inequality in one variable.
- Identify a numerical value(s) that is part of the solution set of a given inequality.

A.5 The student will
a) solve multistep linear inequalities in one variable algebraically and represent the solution graphically; b) represent the solution of linear inequalities in two variables graphically; c) solve practical problems involving inequalities; and d) represent the solution to a system of inequalities graphically.
- Solve multistep linear inequalities in one variable algebraically and represent the solution graphically. (a)
- Apply the properties of real numbers and properties of inequality to solve multistep linear inequalities in one variable algebraically. (a)
- Represent the solution of a linear inequality in two variables graphically. (a)
- Solve practical problems involving linear inequalities. (c)
- Determine whether a coordinate pair is a solution of a linear inequality or a system of linear inequalities. (c)
- Represent the solution of a system of two linear inequalities graphically. (d)
- Determine and verify algebraic solutions using a graphing utility. (a, b, c, d)

All.3 The student will solve
a) absolute value linear equations and inequalities;
- Solve absolute value linear equations or inequalities in one variable algebraically. (a)
- Represent solutions to absolute value linear inequalities in one variable graphically. (a)
### ALGEBRAIC EXPRESSIONS

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| 7.11 The student will evaluate algebraic expressions for given replacement values of the variables.  
  - Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles.  
  - Use the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value | | | | |
| 8.14 The student will  
a) evaluate an algebraic expression for given replacement values of the variables; and  
b) simplify expressions in one variable.  
  - Use the order of operations and apply the properties of real numbers to evaluate algebraic expressions for the given replacement values of the variables. Exponents are limited to whole numbers and bases are limited to integers. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression.  
  - Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles. (a)  
  - Simplify algebraic expressions in one variable. Expressions may need to be expanded (using the distributive property) or require combining like terms to simplify. Expressions will include only linear and numeric terms. Coefficients and numeric terms may be rational. (b) | | | | |
| A.1 The student will  
a) represent verbal quantitative situations algebraically; and  
b) evaluate algebraic expressions for given replacement values of the variables.  
  - Translate between verbal quantitative situations and algebraic expressions and equations. (a)  
  - Represent practical situations with algebraic expressions in a variety of representations (e.g., concrete, pictorial, symbolic, verbal). (a)  
  - Evaluate algebraic expressions, using the order of operations, which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator. (b) | | | | |
| A.2 The student will perform operations on polynomials, including  
a) applying the laws of exponents to perform operations on expressions;  
b) adding, subtracting, multiplying, and dividing polynomials; and  
c) factoring completely first- and second-degree binomials and trinomials in one variable.  
  - Simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents. (a)  
  - Model sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial and symbolic representations. (b) | | | | |
| All.1 The student will  
a) add, subtract, multiply, divide, and simplify rational algebraic expressions;  
b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and  
c) factor polynomials completely in one or two variables.  
  - Add, subtract, multiply, and divide rational algebraic expressions. (a)  
  - Simplify a rational algebraic expression with monomial or binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. (a)  
  - Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions. (a)  
  - Simplify radical expressions containing positive rational numbers and variables. (b)  
  - Convert between radical expressions and expressions containing rational exponents. (b)  
  - Add and subtract radical expressions. (b)  
  - Multiply and divide radical expressions. Simplification may include rationalizing denominators. (b)  
  - Factor polynomials in one or two variables with no more than four terms completely over the set of integers. Factors of the polynomial should be constant, linear, or quadratic. (c)  
  - Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials. (c) | | | | |
• Determine sums and differences of polynomials. (b)
• Determine products of polynomials. The factors should be limited to five or fewer terms (i.e., \((4x + 2)(3x + 5)\) represents four terms and \((x + 1)(2x^2 + x + 3)\) represents five terms). (b)
• Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor. (b)
• Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors. (c)
• Factor and verify algebraic factorizations of polynomials with a graphing utility. (c)

A.3 The student will simplify
a) square roots of whole numbers and monomial algebraic expressions;
b) cube roots of integers; and
c) numerical expressions containing square or cube roots.
• Express the square root of a whole number in simplest form. (a)
• Express the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. (a)
• Express the cube root of an integer in simplest form. (b)
• Simplify a numerical expression containing square or cube roots. (c)
• Add, subtract, and multiply two monomial radical expressions limited to a numerical radicand. (c)

All.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of \(i\).
• Recognize that the square root of \(-1\) is represented as \(i\).
• Simplify radical expressions containing negative rational numbers and express in a \(a + bi\) form.
• Simplify powers of \(i\).
• Add, subtract, and multiply complex numbers.
Making Connections Between and Among Representations in 2016 Mathematics SOL Grades 6 – Algebra II

Virginia Council of Teachers of Mathematics
March 9, 2018

Tina Mazzacane, Mathematics Coordinator
Virginia Department of Education
Office of Science, Technology, Engineering, and Mathematics
Agenda

• Welcome and Introductions
• 2016 Mathematics SOL Implementation Overview
• Effective Mathematics Teaching Practices
• Using and Connecting Math Representations in the 2016 SOL
  – Proportional Relationships to Linear Functions – SOL 6.12 and 7.10
  – Discontinuity – SOL AII.7 (Desmos)
• Support for Implementation
• Closure and Reflection
Welcome and Introductions

- Name
- School Division/School
- Grade Level/Course(s) Taught
2016 Mathematics Standards of Learning Implementation Overview
Implementation of the 2016 Mathematics Standards of Learning - Revised

2017-2018 School Year – Crosswalk Year

- 2009 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning* are included in the written and taught curricula.
- Fall 2017 Standards of Learning assessments measure the 2009 *Mathematics Standards of Learning* and will **not** include field test items measuring the 2016 *Mathematics Standards of Learning*.
- Spring 2018 Standards of Learning assessments measure the 2009 *Mathematics Standards of Learning* and will include field test items measuring the 2016 *Mathematics Standards of Learning*.

2018-2019 School Year – Full-Implementation Year

- 2016 Mathematics Standards of Learning are included in the written and taught curricula. **2009 Mathematics Standards of Learning and 2016 Mathematics Standards of Learning** are included in the written and taught curricula in classrooms administering Fall 2018 Standards of Learning assessments.
- **Fall 2018 Standards of Learning assessments, including End-of-Course (Algebra I, Geometry, and Algebra II), will measure the 2009 Mathematics Standards of Learning and include field test items measuring the 2016 Mathematics Standards of Learning.**
- Spring 2019 (Grades 3-8 and End-of-Course) Standards of Learning assessments measure the 2016 Mathematics Standards of Learning.
Foci of Revisions

- Improve Vertical Progression of Mathematical Content
- Ensure Developmental Appropriateness of Student Expectations
- Increase Support for Teachers in Mathematics Content
- Ensure Proficiency of Elementary Students in Computational Skills
- Improve Precision and Consistency in Mathematical Language and Format
- Clarify Expectations for Teaching and Learning
- 2016 REVISIONS
Mathematics Process Goals for Students

“The content of the mathematics standards is intended to support the five process goals for students”
- 2009 and 2016 Mathematics Standards of Learning
2016 SOL Implementation Support Resources

• 2016 Mathematics Standards of Learning
• 2016 Mathematics Standards Curriculum Frameworks
• 2009 to 2016 Crosswalk (summary of revisions) documents
• 2016 Mathematics SOL Video Playlist (Implementation and Resources)
• Narrated 2016 SOL Summary PowerPoints
• 2017 SOL Mathematics Institute Resources
# Grade 6 – Crosswalk (Summary of Revisions): 2016 Mathematics Standards of Learning and Curriculum Framework

## Additions (2016 SOL)

- 6.6a, b – Operations with two integers and solve practical problems [Moved from 7.3]
- 6.9 EKS – Identify regular polygons; draw lines of symmetry for regular polygons
- 6.11b – Determine the effect on measures of center when a value is added, removed, or changed [Moved from 5.16 EKS]
- 6.12 – Represent proportional relationships between two quantities; determine unit rates and complete ratio tables; determine whether a proportional relationship exists; and make connections among representations of proportional relationships
- 6.13 – Solve practical problems with one-step linear equations
- 6.13 EKS – Write verbal expressions and sentences as algebraic expressions and equations; write algebraic expressions and equations as verbal expressions and sentences
- 6.14 – Represent practical situations with inequalities; solve one-step inequalities involving addition and subtraction, and graph solutions on a number line
- 6.14 EKS – Identify a value that is a solution to an inequality

## Deletions from Grade 6 (2009 SOL)

- 6.9 – Ballpark comparisons between U.S. Customary system and metric system of measurements [Included in 7.3 EKS]
- 6.10d – Describe and determine the volume and surface area of a rectangular prism [Included in 7.4a]
- 6.13 – Properties of quadrilaterals [Included in 7.6a]
- 6.15b – Decide which measure of center is appropriate for a given purpose
- 6.16 – Dependent and independent events [Moved to 8.11a]
- 6.16b – Determine probabilities [Included in 8.11b]
- 6.17 – Arithmetic and geometric sequences [Included in AFDA.1 EKS, All.5]

## Parameter Changes/Clarifications (2016 SOL)

- 6.2a EKS – Equivalencies limited to fractions with denominators of 12 or less or factors of 100
- 6.2b – Compare and order fractions, decimals, and percents extended to positive rational numbers; EKS limited to no more than four; EKS limited to fractions with denominators of 12 or less or factors of 100 to include proper, improper and mixed numbers
- 6.4 EKS – Limitation changed to whole number exponents, versus natural number exponents
- 6.5c EKS – Divisors limited to 3 digit number and decimal divisors limited to hundredths
- 6.6c – Simplify numerical expressions [Moved and modified from 6.8] extended to include integers [EKS extended to include absolute value; exponents limited to 1, 2, and 3 and bases limited to whole numbers; expression may have no more than 3 operations]
- 6.8b EKS - Coordinate values limited to integers
- 6.10a EKS – Number of data values represented in a circle graph limited in order to make comparisons that have denominators of 12 or less or those that are factors of 100
- 6.10c – Compare circle graphs with other graphs now specified as bar graphs, pictographs, and line plots
- 6.11a EKS – Represent mean as a balance point graphically on a line plot
- 6.13 EKS – Solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.
- 6.6, 6.13, 6.14 EKS and US – Apply properties of real numbers and properties of equality/inequality

## Moves within Grade 6 (2009 SOL to 2016 SOL)

- 6.2a – [Moved to 6.2 EKS]
- 6.2 b, c – [Included in 6.2a]
- 6.2d – Compare and order fractions, mixed numbers, decimals and percents [Included in 6.2b]
- 6.4 – [Moved to 6.5a EKS]
- 6.5 – [Moved to 6.4]
- 6.6 – [Moved to 6.5a, b]
- 6.7 – [Moved to 6.5c]
- 6.8 – [Moved to 6.6c and modified]
- 6.10a, b, c – [Moved to 6.7a, b, c]
- 6.11 – [Moved to 6.8]
- 6.12 – [Moved to 6.9]; Draw polygons in the coordinate plane and find side lengths using the coordinates [Moved to 6.8]
- 6.13 – [Moved to 6.12]
- 6.14 – [Moved to 6.10]
- 6.15a – [Moved to 6.11a]
- 6.18 – [Moved to 6.13]
- 6.19 – [Moved to 6.13]
- 6.19 – Investigate and recognize properties [Incorporated into EKS and US for 6.6, 6.13, and 6.14]
- 6.20 – [Moved to 6.14]

EKS = Essential Knowledge and Skills, referring to the column on the right side of the Curriculum Framework
US = Understanding the Standard, referring to the column on the left side of the Curriculum Framework
### Comparison of Mathematics Standards of Learning – 2009 to 2016

<table>
<thead>
<tr>
<th>2009 SOL</th>
<th>2016 SOL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Number Sense</strong></td>
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<tr>
<td><em>On the state assessment, items measuring this objective are assessed without the use of a calculator.</em></td>
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</tr>
<tr>
<td><strong>6.1</strong> The student will describe and compare data, using ratios, and will use appropriate notations, such as ( \frac{a}{b} ), ( a ) to ( b ), and ( a:b ).</td>
<td><strong>6.1</strong> The student will represent relationships between quantities using ratios, and will use appropriate notations, such as ( \frac{a}{b} ), ( a ) to ( b ), and ( a:b ).</td>
</tr>
</tbody>
</table>
| **6.2** The student will  
  a) investigate and describe fractions, decimals, and percents as ratios; [Moved to 6.2 EKS]  
  b) identify a given fraction, decimal, or percent from a representation; [Included in 6.2a]  
  c) demonstrate equivalent relationships among fractions, decimals, and percents;* and [Included in 6.2a]  
  d) compare and order fractions, decimals, and percents.* [Included in 6.2b] | **6.2** The student will  
  a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents;*  
  b) compare and order positive rational numbers.* |
| **6.3** The student will  
  a) identify and represent integers;  
  b) order and compare integers; and  
  c) identify and describe absolute value of integers. | **6.3** The student will  
  a) identify and represent integers;  
  b) compare and order integers; and  
  c) identify and describe absolute value of integers. |
| **6.4** The student will demonstrate multiple representations of multiplication and division of fractions. [Moved to 6.5 EKS] | **6.4** The student will recognize and represent patterns with whole number exponents and perfect squares. |
| **6.5** The student will investigate and describe concepts of positive exponents and perfect squares. | **6.5** The student will  
  a) multiply and divide fractions and mixed numbers;*  
  b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.  
  c) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and  
  c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals. [Moved from 6.7] |
| **6.6** The student will  
  a) multiply and divide fractions and mixed numbers;* and  
  b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions. | **6.6** The student will  
  a) multiply and divide fractions and mixed numbers;* and  
  b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions. |
Effective Mathematics Teaching Practices
“Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.”

NCTM (2014), *Principles to Action*
Executive Summary

YOU make all the difference! Thank you!!!
### NCTM Principles to Actions

*Ensuring Mathematical Success for All*

<table>
<thead>
<tr>
<th>High Leverage Mathematics Teaching Practices</th>
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<td>1. Establish mathematics goals to focus learning.</td>
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<tr>
<td>2. Implement tasks that promote reasoning and problem solving.</td>
</tr>
<tr>
<td>3. Use and connect mathematical representations.</td>
</tr>
<tr>
<td>4. Facilitate meaningful mathematical discourse.</td>
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<tr>
<td>5. Pose purposeful questions.</td>
</tr>
<tr>
<td>6. Build procedural fluency from conceptual understanding.</td>
</tr>
<tr>
<td>7. Support productive struggle in learning mathematics.</td>
</tr>
<tr>
<td>8. Elicit and use evidence of student thinking.</td>
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**NCTM Principles to Actions**

*Ensuring Mathematical Success for All*

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Use and Connect Mathematical Representations

<table>
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<tr>
<th>Practice 3. Use and connect mathematical representations.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher Actions</strong></td>
</tr>
<tr>
<td>Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
</tr>
<tr>
<td>Introducing forms of representations that can be useful to students.</td>
</tr>
<tr>
<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
</tr>
<tr>
<td>Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
</tr>
<tr>
<td>Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
</tr>
<tr>
<td><strong>Student Actions</strong></td>
</tr>
<tr>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
</tr>
<tr>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
</tr>
<tr>
<td>Sketching diagrams to make sense of problem situations.</td>
</tr>
<tr>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
</tr>
<tr>
<td>Considering the advantages or suitability of using various representations when solving problems.</td>
</tr>
</tbody>
</table>

Use and Connect Mathematical Representations

The term representation refers both to process and to product – in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself. Moreover, the term applies to processes and products that are observable externally as well as to those that occur “internally” in the minds of people doing mathematics.

- NCTM 2000, p. 67
Teaching Framework for Mathematics

Establish mathematics goals to focus learning.


Implement tasks that promote reasoning and problem solving.

Build procedural fluency from conceptual understanding.

Facilitate meaningful mathematical discourse.

Use and connect mathematical representations.

Pose purposeful questions.

Elicit and use evidence of student thinking.

Support productive struggle in learning mathematics.

Virginia Mathematics Process Goals

1. Representations
2. Connections
3. Problem Solving
4. Reasoning
5. Communication
Using and Connecting Mathematical Representations in the 2016 SOL
Mathematical Representations - 2016 SOL

6.12 The student will
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

7.10 The student will
e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

8.16 The student will
e) make connections between and among representations of a linear function using verbal descriptions, tables, equation, and graphs.
<table>
<thead>
<tr>
<th></th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional Relationships/Functions</td>
<td>Represent proportional relationships; unit rates; ratio tables; determine if a proportional relationship exists; and make connections between and among representations</td>
<td>Slope as a rate of change; write an equation in $y = mx$ form to represent a proportional relationship and $y = x + b$ form to represent an additive relationship and graph their lines; and make connections between and among representations</td>
<td>Slope of a line given a graph; slope and $y$-intercept of a linear function and graph using $y = mx + b$ form; and make connections between and among representations</td>
</tr>
<tr>
<td>Solving Equations</td>
<td>Solve one-step equations including within practical problems</td>
<td>Solve two-step equations including within practical problems</td>
<td>Solve multi-step equations (including variables on both sides) including within practical problems</td>
</tr>
<tr>
<td>Solving Inequalities</td>
<td>Represent practical situations with inequalities; solve one-step inequalities using addition/subtraction and graph the solution on a number line</td>
<td>Solve one- and two-step inequalities (all operations) including within practical problems; graph solution on a number line</td>
<td>Solve multi-step inequalities (including variables on both sides) including within practical problems; graph solution on a number line</td>
</tr>
<tr>
<td>Algebraic Expressions</td>
<td>Evaluate algebraic expressions for given replacement values of the variable</td>
<td>Evaluate an algebraic expression for given replacement values of the variable; simplify expressions in one variable</td>
<td></td>
</tr>
</tbody>
</table>
## Algebra (Proportional Reasoning) Progression

<table>
<thead>
<tr>
<th>SOL 6.12</th>
<th>SOL 7.10</th>
<th>SOL 8.16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Essential Knowledge and Skills</strong></td>
<td><strong>Essential Knowledge and Skills</strong></td>
<td><strong>Essential Knowledge and Skills</strong></td>
</tr>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:</td>
</tr>
<tr>
<td>- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)</td>
<td>- Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope; or a rate of change, slope will be limited to positive values. (a)</td>
<td>- Given a table of values for a linear function, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)</td>
</tr>
<tr>
<td>- Identify the unit rate of a proportional relationship represented by a table, or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)</td>
<td>- Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b)</td>
<td>- Given a linear function in the form ( y = mx + b ), identify the slope and y-intercept. (b)</td>
</tr>
<tr>
<td>- Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c)</td>
<td>- Determine the y-intercept, ( b ), in a linear equation, given a practical situation, and write an equation in the form ( y = mx - b ) to represent the relationship. (c)</td>
<td>- Given the equation of a linear function in the form ( y = mx + b ), graph the function. The value of the y-intercept will be limited to integers. (d)</td>
</tr>
<tr>
<td>- Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)</td>
<td>- Graph a line representing an additive relationship ( y = x + b ), given an ordered pair on the line and the slope; or a rate of change, slope will be limited to positive values. (c)</td>
<td>- Make connections between and among representations of a linear function graphically and algebraically in a practical situation in which the slope, ( m ), and y-intercept are described verbally. (e)</td>
</tr>
<tr>
<td>- Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)</td>
<td>- Graph a line representing a non-proportional or additive relationship ( y = mx + b ) between two quantities using verbal descriptions, tables, equations, and graphs. (e)</td>
<td>- Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e)</td>
</tr>
</tbody>
</table>
Work with a partner to create a model using linking cubes to solve this problem.

Perfect Purple Paint

- Perfect Purple Paint is made by mixing 2 cups blue paint to 3 cups red paint.

- How much of each color would be needed to make a total of 20 cups of Perfect Purple Paint?
Introduction to Ratios and Proportional Relationships


• Using the list of teacher and student actions for Mathematics Teaching Practice #3: Use and Connect Mathematical Representations:
  o Identify teacher actions from the video that promote using and connecting representations?
  o Identify student actions from the video that promote using and connecting representations?
UNIT RATE: 1 ½ CUPS RED PER 1 CUP BLUE
# RATIO TABLE

<table>
<thead>
<tr>
<th>BLUE (cups)</th>
<th>RED (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{2}$ or 1.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

\[
y = \frac{3}{2}x
\]

\[
y = mx
\]
Functions, Discontinuity, and Desmos
Multiple Representations of Functions

A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically including

f) connections between and among multiple representations of functions including verbal descriptions, tables, equations, and graphs.

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

g) connections between and among multiple representations of functions including verbal descriptions, tables, equations, and graphs.
SOL AII.7 - Discontinuity

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include:

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

<table>
<thead>
<tr>
<th>Understanding the Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A function is said to be continuous on an interval if its graph has no jumps or holes in that interval.</td>
</tr>
<tr>
<td>• The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.</td>
</tr>
<tr>
<td>• Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Essential Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. (a, d, e)</td>
</tr>
<tr>
<td>• Describe a function as continuous or discontinuous. (a)</td>
</tr>
</tbody>
</table>
Desmos - Open Source Versions

DESMOS ONLINE GRAPHING UTILITY: https://desmos.com/calculator

DESMOS CLASSROOM ACTIVITIES: https://teacher.desmos.com

DESMOS STUDENT ACTIVITY LINK: https://student.desmos.com

LEARN MORE ABOUT DESMOS: https://learn.desmos.com
Desmos – Additional Features in Development

DESMOS GEOMETRY UTILITY (Beta Version): https://desmos.geometry.com/

DESMOS GRAPHING UTILITY ENHANCEMENTS https://www.desmos.com/calculator:
  DISTRIBUTIONS – adds statistical tests, such as normal cdf and pdf, t-tests, etc. to graphing utility functions

ADDITIONAL ACCESSIBILITY FEATURES
Desmos – Graphing Utility

https://www.desmos.com/calculator

\[ f(x) = 2x - 3 \quad \text{for} \quad x < 2 \]

\[ f(x) = -4 \quad \text{for} \quad x \geq 2 \]
Card Sort – Making Connections (SOL 7.10e)

Welcome!

Enter your class code:

Join

Sign in to come back to your work later:

Google Sign in with Google or Sign in with Desmos

Hey, students!

Go to student.desmos.com and type in:

24UYH

You can also share this link with your students:

https://student.desmos.com/?prepopulateCo
Implementation Support Resources
Implementation Support Resources

Currently Available

• 2016 Mathematics Standards of Learning
• 2016 Mathematics Standards Curriculum Frameworks
• 2009 to 2016 Crosswalk (summary of revisions) documents
• 2016 Mathematics SOL Video Playlist (Overview, Vertical Progression & Support, Implementation and Resources)
• Progressions for Selected Content Strands (K-5: Number and Number Sense, Computation and Estimation)
• Narrated 2016 SOL Summary PowerPoints
• 2017 SOL Mathematics Institutes Professional Development Resources

Coming in Spring/Summer 2018

• Updated and New Lesson Plans
• Updated Vocabulary Cards
• Virginia Board of Education Textbook Approval List
Closure and Participant Reflection

Exit Ticket – What Stuck?

On a Post-It Note –
This is what stuck with me today........
QUESTIONS?

Tina.Mazzacane@doe.virginia.gov
Mathematics@doe.virginia.gov
Making Connections

Reporting Category: Patterns, Functions, and Algebra
Topic: Determine Slope
Primary SOL: 7.10 The student will
 e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.
Related SOL: 7.10 a – d

Materials
• Activity Sort (attached pages 3 - 8)
• Scientific Calculator (optional)

Vocabulary
 Proportional, relationship, constant of proportionality, unit rate, multiplicative relationship, additive relationship (earlier grades)
 slope, rate of change, constant ratio, slope triangle, y-intercept (7.10)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)
1. Students should review information on ratio tables, multiplicative relationships, equivalent ratios, proportional relationships, slope \( m \), additive relationships, and y-intercept \( b \).
2. After the teacher prints and cuts out the graphs, tables, equations, and verbal situations from the Activity Sort; then distribute to students. The students will work in pairs or groups to match each verbal situation with the correct graph, table, and equation.
3. Students should correctly match the representations, then create two separate piles. One pile should represent those that are additive, while the other represents those that are proportional.
4. Students should then discuss the similarities and differences between the two piles that were created.
5. Students should notice that some of the graphs only include the points, but others connect the points with a line. Ask students why this might be the case. Students should note that the graphs connected with lines represent situations in which there are an infinite number of points that could be graphed (e.g. comparing time and distance) whereas others only have specific points that could represent the situation (e.g. weeks taking lessons and the number of songs that can be played). Also note with students that all of the graphs are represented by points in the first quadrant only \( x \geq 0 \). Since the graphs represent practical situations, there is a limitation on the values that should be included in the graph. Whereas, if we were just graphing an equation, such as \( y = 3x \), without having a context for the relationship, we would connect the points with a straight line and there would be no limitation on the values of the variables.
Assessment

- **Questions**
  - How can you use a table of values to determine the graph of a line?
  - How can you use the graph of a line to determine an equation of a line?
  - What are key words in the real world examples that help you determine if the situation is additive or multiplicative?

- **Journal/Writing Prompts**
  - What is the difference between an additive and multiplicative relationship in a table of values?
  - What is the difference between an additive and multiplicative relationship in the graphs of the lines?

- **Other**
  - Students can create their own situation and share it with another student to graph, put in a table and make an equation that matches the practical situation.

Extensions and Connections (for all students)

- Students can enter in several \( y = x + b \) and \( y = mx \) equations on a graphing calculator or desmos graphing calculator in order to make connections to the \( y \)-intercept of a line and the value of \( b \).

Strategies for Differentiation

- Students can utilize the sort as a jigsaw activity – distributing/grouping the cards based on student ability.
- Review prior vocabulary and pre-teach new essential vocabulary to certain students as necessary prior to the lesson introduction.
- Ensure that students of varying abilities are represented in each small group, and that all students have a meaningful role.
Activity Sort
Desmos Digital Version: https://teacher.desmos.com/activitybuilder/custom/5a9363f490db701bb48ad9ee

Proportional Relationship #1:

Mary and her friends set out to sea on their annual fishing trip. Their distance from the shore in miles, $y$, increases by 3 miles each hour, $x$. Write an equation to model this relationship.

$$y = 3x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>
Proportional Relationship #2:

Shawn has started a small business selling bat houses. He can build 6 houses in 4 hours. Write an equation to model the relationship between the number of hours Shawn has worked, $x$, and the number of houses, $y$, he has built.

\[
y = \frac{3}{2}x
\]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>
Proportional Relationship #3:

Martha can swim 12 yards in 6 seconds. Write an equation to represent the number of yards, $y$, Martha can swim in $x$ seconds.

$$y = 2x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Additive Relationship #1:

\[ y = x + 3 \]

Sam is taking guitar lessons and will learn one new song per week. When he started, he already knew how to play 3 songs. Write an equation to model the relationship between the number of songs Sam knows how to play, \( y \), and the number of weeks, \( x \), he has been taking lessons.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>
Additive Relationship #2:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Every issue of Billy’s favorite comic book has 4 pages of ads. The remaining pages contain the story. Write an equation that relates the total number of pages in a comic book, $x$, and the number of story pages, $y$.

$y = x - 4$
Additive Relationship #3:

On each necklace Sarah makes, there are 5 more purple beads than silver beads. Write an equation to represent the relationship between the number of silver beads, $x$, and the number of purple beads, $y$.

$$y = x + 5$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>
Functions: Domain, Range, Continuity and End Behavior

Strand: Functions
Topic: Finding domain and range, continuity, and end behavior for a given function

Primary SOL: AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include
   a) domain, range, and continuity; and
   h) end behavior.

Related SOL: AII.6

Materials
- Activity sheets 1-3 (attached)
- Graphing utility
- Demonstration tool (document camera, graphing software, or other)

Vocabulary
domain, range, input, output, independent variable, dependent variable, negative infinity, positive infinity, interval notation, end behavior, discontinuous, continuous, vertical asymptote, horizontal asymptote, non-removable discontinuity, removable discontinuity

Student/Teacher Actions – What should students be doing? What should teachers be doing?

Timeframe: 90 minutes

1. The first activity for this lesson involves the teacher demonstrating how to graph certain functions and discussing how to find the domain, range, continuity, and end behavior. Students can use Activity Sheet 1: Function Characteristics Examples handout to record their notes during the class discussion if needed.

2. Have students graph the first example function, \( f(x) = x^2 \), while you demonstrate the graphing steps with a graphing utility. Students will benefit from seeing graphs displayed with different graphing utilities. Have students sketch their graph on the Activity Sheet 1. Then, have students discuss with partners the definitions of domain and range and determine the domain and range of the quadratic function in example one.

3. Discuss the difference between a continuous function and a discontinuous function. Have students determine whether or not the quadratic function in example one is a continuous function.

4. Demonstrate, and have students copy into notes, how to express the domain \( \{x \mid x \in \mathbb{R}\} \) and range \( \{f(x) \mid f(x) \geq 0\} \) in both set notation and interval notation. In interval notation, the domain is \((-\infty, \infty)\) and the range is \([0, \infty)\).
5. Discuss the end behavior of the function, both as $x$ approaches negative infinity and as it approaches positive infinity. Demonstrate the proper notation for expressing end behavior. $(x \to \infty, f(x) \to \infty)$ and $(x \to -\infty, f(x) \to \infty)$

6. Have students use a graphing utility to graph example 2, $f(x) = \sqrt{2x - 6}$, as you graph it. Use a demonstration tool (document camera, calculator software, other) to display the graph, which is visible to the class. Ask students, “Why do we see the graph in the first quadrant only? How would the domain and range be different from the parabola?” Use think-pair-share to allow students to think about their answer, share it with a partner, and then share with the class. Have students sketch the graph and write the domain and range in both set and interval notation.

7. Ask students about the continuity of the function. You should note for students that the function is continuous over its domain. Ask students, “What about the end behavior? What happens to the function as $x$ approaches infinity? What about as $x$ approaches negative infinity?” Clarify for students that the function does not have end behavior as $x$ approaches negative infinity since that is not part of its domain.

8. Have students use a graphing utility to graph example 3, $f(x) = \frac{x}{x + 2}$, as you graph it. Use a demonstration tool (document camera, calculator emulator, or other) to display the graph, which is visible to the class. Ask students, “What do you notice about this function?” Students should mention that the graph seems to be in pieces. Ask students, “What does this mean about the continuity of the function?” Students should recognize that the function is discontinuous. Tell students that this is an example of non-removable discontinuity.

9. Ask students, “Where is there a gap in the pieces of the function?” Students should recognize that there is a gap in the function at $x = -2$. Ask students to think about why there might be a gap there. “What about the function might cause it to have a gap at $x = -2$?” Students should notice that $x = -2$ would make the denominator 0 and would cause the function to be undefined there. Ask students, “How would this affect the domain of this function?” Identify this as the vertical asymptote of the function. Model how to write the domain of the function for students in both interval and set notation.

10. Ask students if they see any other possible asymptote lines. Students should respond with the horizontal asymptote line at $y = 1$. Identify that as the horizontal asymptote. Ask students, “Why is there a gap at $y = 1$? Does this change the domain or the range?” Have students write the range and check it with a partner. Discuss why there might be a gap at $y = 1$. Set the function equal to 1 and have students try to solve the equation. (While asymptote lines may be covered before or after this lesson, this is a good opportunity to introduce or review that concept, particularly for rational functions.)

11. Ask students to identify the end behavior of this function for example 3. Remind students that the end behavior is what happens as $x$ approaches positive or negative infinity and not what happens in the middle of the graph. Ask students, “What do you notice about the end behavior and the horizontal asymptote?”
12. Show students a graph of the function, \( f(x) = \begin{cases} 
2x - 3, & x < 2 \\
-4, & x \geq 2 
\end{cases} \) as shown below and example four on Activity Sheet 1: Function Characteristics Examples.

![Graph](image)

Ask students to write the domain and range, end behavior, and describe whether or not it is continuous. Students should notice that the domain is all real numbers but the range is actually over two different intervals. The range in set notation is \( \{ y \mid y < 1 \text{ or } y = 4 \} \) and in interval notation \( ( -\infty, 1 ) \cup [ 4 ] \). Point out to students that one of the endpoints is open while the other is closed. Students should recognize that this function is discontinuous. Define this as removable discontinuity.

13. Distribute copies of the Activity Sheet 2: Where to Begin and End handout and have students work in pairs or groups to complete it.

14. Distribute copies of the Activity Sheet 3: Domain and Range handout for review the next day.

Assessment

- **Questions**
  - What are the domain and range for \( f(x) = \sqrt{x - 4} \)?
  - How would you describe the end behavior of \( f(x) = 2^x \)?
  - Draw a function that has a non-removable discontinuity at \( x = 3 \).

- **Journal/writing prompts**
  - Create and describe a function, both algebraically and graphically, that would have end behavior of \( x \to -\infty, f(x) \to -\infty \) and \( x \to \infty, f(x) \to -\infty \), domain of \( (-\infty, \infty) \) and range of \( (-\infty, 5] \).
  - Discuss whether it is possible for a function to have a removable discontinuity at \( x = -3 \) and still have a domain of all real numbers. Justify your answer in at least two ways.
  - Compare and contrast the end behaviors of a quadratic function and its reflection over the x-axis.
• Other Assessments
  o Draw a function with a domain of \([-4,1]\) and a range of \([2,6]\).

Extensions and Connections
• Have students state the domain and range for a circle with center \((2,5)\) and radius 4.
• Present the following problem and discuss the domain and range of the function for this situation.
  The school is planning a trip to visit a college about 3 hours away. They are renting a coach bus that holds 40 students. The administration has determined that it will cost $12.50 per person to rent the bus and a flat fee of $25.00 to park. The function that models this is 
  \[ c(s) = 12.50s + 25 \]
  where \(s\) represents the number of students on the trip and \(c(s)\) represent the cost. What is the domain and range of the function for this situation?

Strategies for Differentiation
• Have student model with their arms the end behavior of a polynomial function, depending on the sign of the leading coefficient and the degree of the polynomial.
• Have students model a given end behavior using their arms.
• Have students cover up the area of the graph not included in the domain or range to help them identify the regions.
  • Introduce domain and range by having students describe graphs of functions to each other using the appropriate vocabulary similar in format to $25,000 or Pictionary with graphs by having students describe functions to each other using the appropriate vocabulary.

Note: The following pages are intended for classroom use for students as a visual aid to learning.
Activity Sheet 1: Function Characteristics Examples

Ex. 1

Domain: Range:

Is the function continuous?

End Behavior

Ex. 2

Domain: Range:

Is the function continuous?

End Behavior

Ex. 3

Domain: Range:

Is the function continuous?

End Behavior
Ex. 4

Is the function continuous?

End Behavior
Activity Sheet 2: Where to Begin and End

For each function below, state the domain and range, end behavior, and continuity.

1. [Graph]
   - Domain: __________________________
   - Range: __________________________
   - End Behavior: ____________________
   - Describe the continuity of the function: ____________________

2. [Graph]
   - Domain: __________________________
   - Range: __________________________
   - End Behavior: ____________________
   - Describe the continuity of the function: ____________________

3. [Graph]
   - Domain: __________________________
   - Range: __________________________
   - End Behavior: ____________________
   - Describe the continuity of the function: ____________________

4. [Graph]
   - Domain: __________________________
   - Range: __________________________
   - End Behavior: ____________________
   - Describe the continuity of the function: ____________________
5. \( f(x) = \frac{2x}{x-1} \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:

6. \( f(x) = -3x + 5 \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:

7. \( f(x) = (x + 3)^2 \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:

8. \( f(x) = -2x^2 - 2 \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:

9. \( f(x) = x^3 + 6x^2 + 9x \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:

10. \( f(x) = \left(\frac{1}{3}\right)^x + 2 \)

Domain: __________________________

Range: __________________________

End Behavior __________________________

Describe the continuity of the function:
Activity Sheet 3: Domain and Range

For each function below, graph the function, state the domain and range, end behavior and continuity.

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
<th>Domain and Range</th>
<th>End Behavior</th>
<th>Continuity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( f(x) = \begin{cases} -3, &amp; x \leq -4 \ x + 1, &amp; x &gt; -4 \end{cases} )</td>
<td><img src="image1" alt="Graph" /></td>
<td>Domain:</td>
<td>Range:</td>
<td></td>
</tr>
<tr>
<td>2. ( y = x^2 + 3 )</td>
<td><img src="image2" alt="Graph" /></td>
<td>Domain:</td>
<td>Range:</td>
<td></td>
</tr>
<tr>
<td>3. ( g(x) = \frac{x}{x + 5} )</td>
<td><img src="image3" alt="Graph" /></td>
<td>Domain:</td>
<td>Range:</td>
<td></td>
</tr>
<tr>
<td>4. ( m(x) = 3^x - 1 )</td>
<td><img src="image4" alt="Graph" /></td>
<td>Domain:</td>
<td>Range:</td>
<td></td>
</tr>
<tr>
<td>6. ( y = \left( \frac{1}{2} \right)^x )</td>
<td><img src="image5" alt="Graph" /></td>
<td>Domain:</td>
<td>Range:</td>
<td></td>
</tr>
</tbody>
</table>